University of Diyala
College of Engineering
Department of Materials

Fundamentals of Electric Circuits

Lecture Six

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2016

2-9 Series Resistors and Voltage Division

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 1. The two resistors are in series, since the same current i flows in both of them.

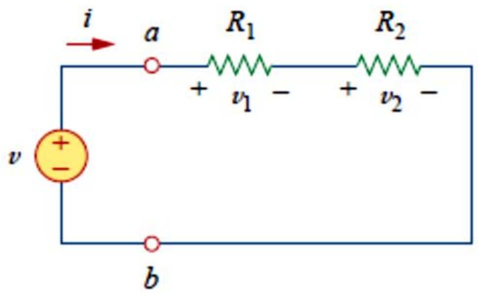

Fig. 1 A single-loop circuit with two resistors in series.

Applying Ohm’s law to each of the resistors, we obtain

$$v_1 = iR_1, \quad v_2 = iR_2 \qquad (1)$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \qquad (2)$$

Combining Eqs. (1) and (2), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \qquad (3)$$

Or

$$i = \frac{v}{R_1 + R_2} \qquad (4)$$

Notice that Eq. (3) can be written as

$$v = iR_{eq} \qquad (5)$$

implying that the two resistors can be replaced by an equivalent resistor R_{eq} ; that is,

$$R_{eq} = R_1 + R_2 \qquad (6)$$

Thus, Fig. 1 can be replaced by the equivalent circuit in Fig. 2. The two circuits in Figs. 1 and 2 are equivalent because they exhibit the same voltage-current relationships at the terminals *a*-*b*. An equivalent circuit such as the one in Fig. 2 is useful in simplifying the analysis of a circuit.

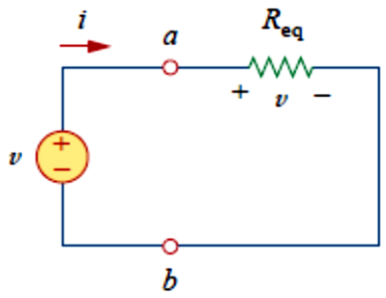

Fig. 2 Equivalent circuit of the Fig. 1 circuit

In general,

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For *N* resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n \qquad (7)$$

To determine the voltage across each resistor in Fig. 1, we substitute Eq. (3) into Eq. (1) and obtain

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v \qquad (8)$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 1 is called a *voltage divider*. In general, if a voltage divider has *N* resistors $(R_1, R_2, \dots, R_N)$ in series with the source voltage v, the *n*th resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v \qquad (9)$$

2-10 Parallel Resistors and Current Division

Consider the circuit in Fig. 3, where two resistors are connected in parallel and therefore have the same voltage across them.

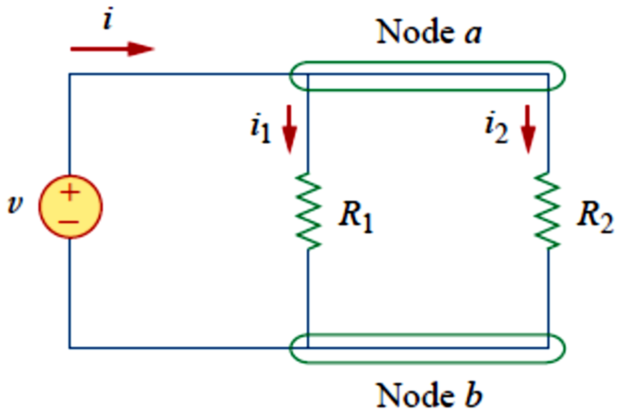

Fig. 3 Two resistors in parallel.

From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

Or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2} \qquad (10)$$

Applying KCL at node *a* gives the total current i as

$$i = i_1 + i_1 \qquad (11)$$

Substituting Eq. (10) into Eq. (11), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \qquad (12)$$

Where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (13)$$

Or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

Or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad (14)$$

Thus,

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (14), if $R_1 = R_2$, then $R_{eq} = R_1/2$.

We can extend the result in Eq. (13) to the general case of a circuit with *N* resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \qquad (15)$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R, then$

$$R_{eq} = \frac{R}{N} \qquad (16)$$

For example, if four 100-Ω resistors are connected in parallel, their equivalent resistance is 25 Ω.

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (15), the equivalent conductance for N resistors in parallel is

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N \qquad (17)$$

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

Thus the equivalent conductance G_{eq} of N resistors in series (such as shown in Fig. 1) is

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{R_N} \qquad (18)$$

Given the total current i entering node a in Fig. 4, how do we obtain current i_1 and i_2? We know that the equivalent resistor has the same voltage, or

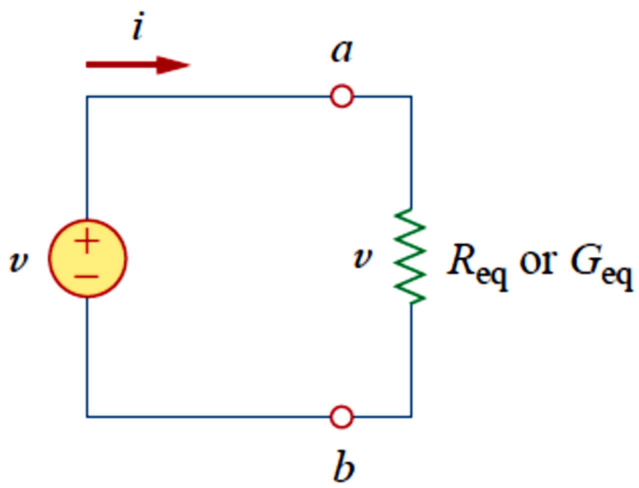

Fig. 4

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \qquad (19)$$

Combining Eqs. (10) and (18) results in

$$i_1 = \frac{R_2 i}{R_1 + R_2}, \qquad i_2 = \frac{R_1 i}{R_1 + R_2} \qquad (20)$$

Example 1: Find R_{eq} for the circuit shown in Fig. 5.

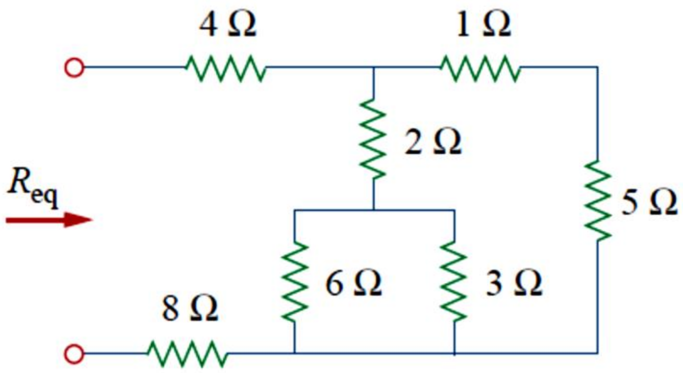

Fig. 5

Example 2: Find R_{eq} for the circuit shown in Fig. 6.

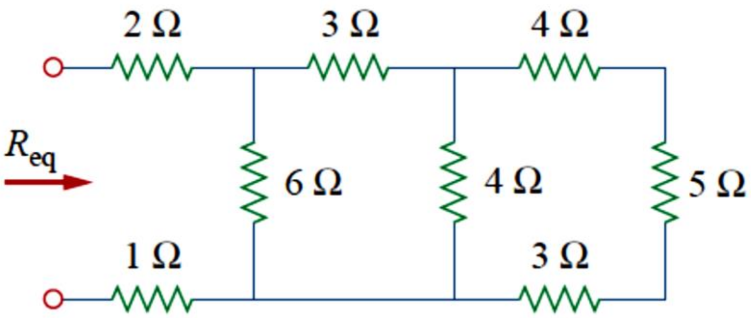

Fig. 6

Example 3: Find R_{eq} for the circuit shown in Fig. 7.

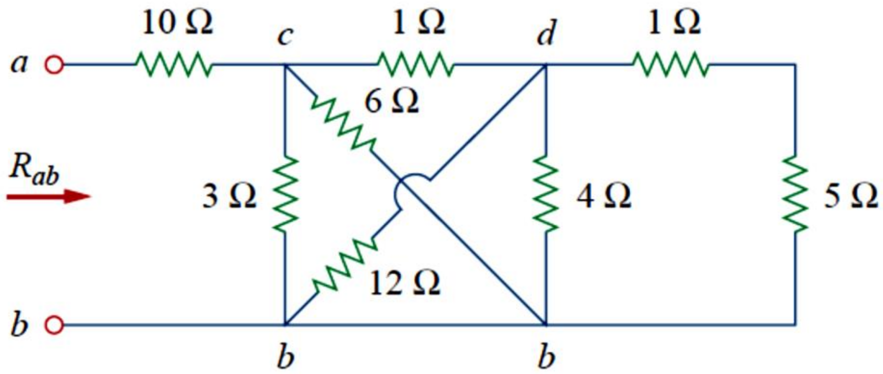

Fig. 7

Example 4: Find R_{ab} for the circuit shown in Fig. 8.

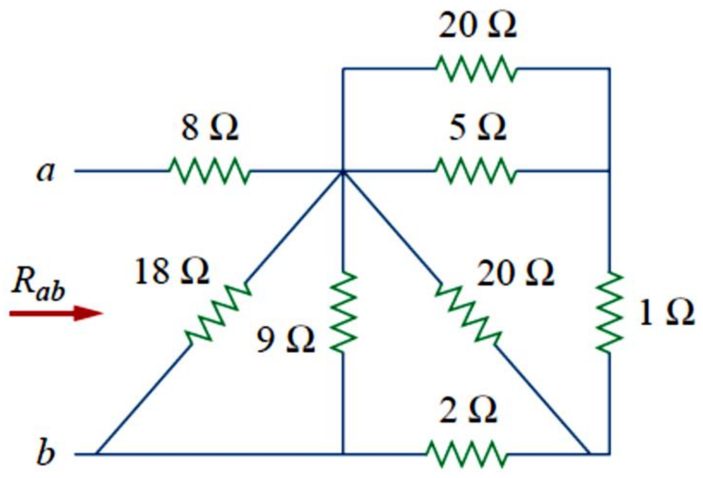

Fig. 8

Example 5: Find G_{eq} for the circuit shown in Fig. 9.

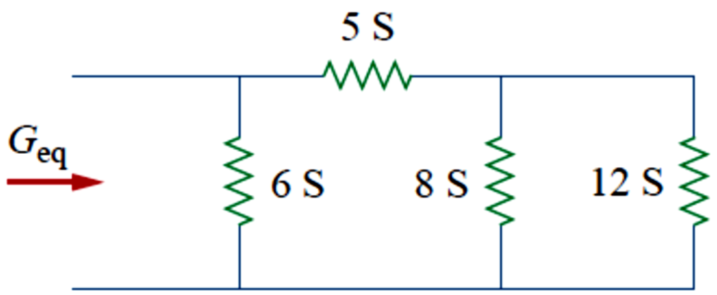

Fig. 9

Example 6: Find G_{eq} for the circuit shown in Fig. 10.

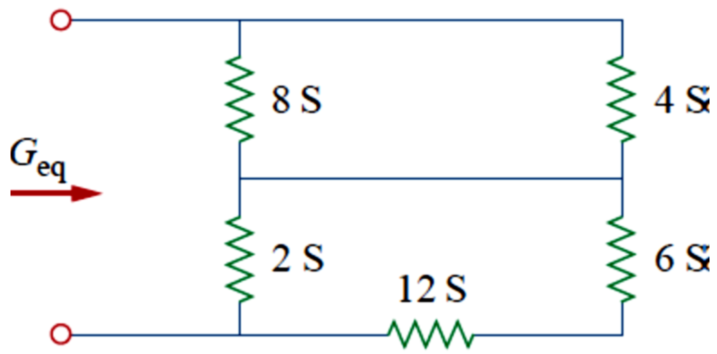

Fig. 10

Example 7: Find i_o and v_o in the circuit shown if Fig. 11. Calculate the power dissipated in the 3Ω resistor.

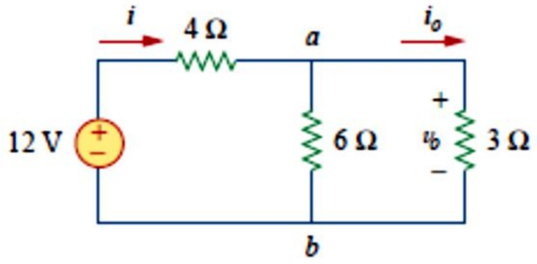

Fig. 11

Example 8: Find v_1 and v_2 in the circuit shown if Fig. 12. Also Calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

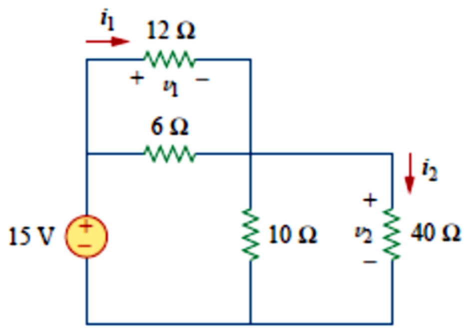

Fig. 12

Example 9: For the circuit shown in Fig. 13, determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

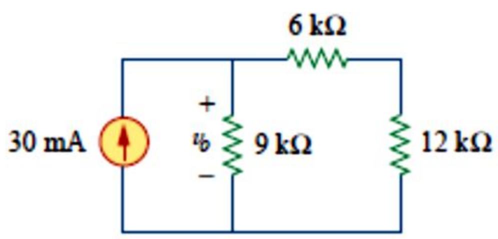

Fig. 13

Example 10: For the circuit shown in Fig. 14, find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ, resistors and (c) the power supplied by the current source.